

# Stepwise discovery of geometrical knowledge in GeoGebra

Zoltán Kovács

The Private University College of Education of the Diocese of Linz, Austria

—  
JKU Linz, Austria

CSGG 2022, 15 September

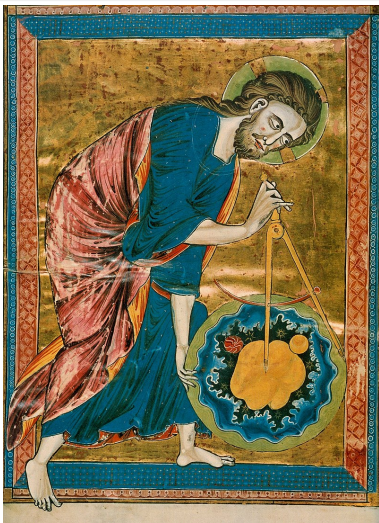
Author is supported by the grant  
PID2020-113192GB-I00 from the Spanish MICINN.

GeoGebra has recently been extended to help discover interesting geometric properties in a planar construction. To collect information on *generally true* properties like **parallelism of lines**, **perpendicularity**, **equality of lengths of segments**, **collinearity** or **conyclicity** of points, were already achievable by using the **Discover** tool or command in an experimental version of GeoGebra, “GeoGebra Discovery”.

The presentation introduces a further improvement: the **stepwise discovery mode** to discover generally true statements *on the fly* (every new point will be checked by the program automatically). We will see how stepwise discovery mode can be helpful in teaching planar geometry at secondary level, or in the researcher’s work.

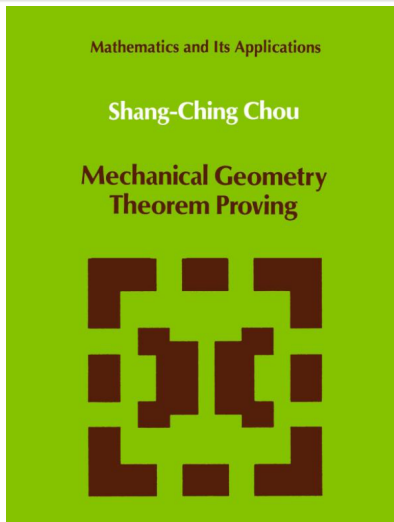
# The beginnings

Codex Vindobonensis 2554 (Paris, ca. 1220-1230); Chou 1987



# The beginnings

Codex Vindobonensis 2554 (Paris, ca. 1220-1230); Chou 1987



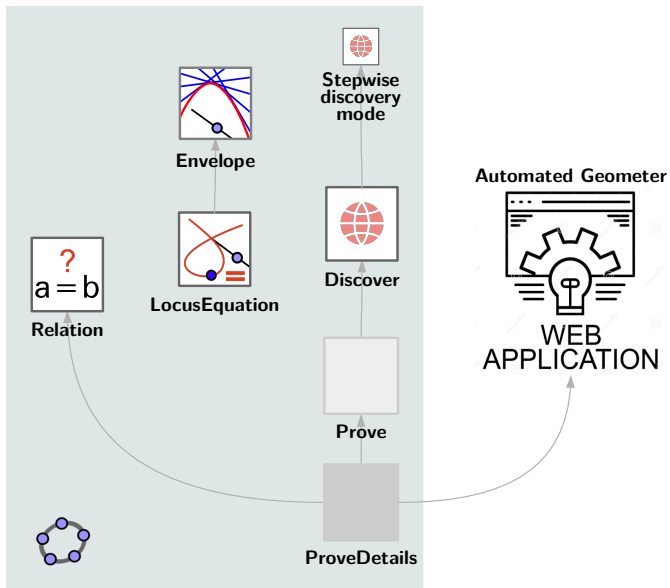
512 theorems in the plane geometry proven mechanically with algebraic means (coordinates, equation systems)

- Algebraization (using coordinates, polynomial equations)
  - Wu-Ritt characteristic sets (fast)
  - Elimination via Gröbner bases (slower but more complete)
  - Real geometry (very slow but complete)
    - cylindrical algebraic decomposition (Mathematica, Tarski)
    - regular chains (Maple)
- Synthetic methods via axioms (very slow but complete)

- Algebraization (using coordinates, polynomial equations)
  - Wu-Ritt characteristic sets (fast)
  - **Elimination via Gröbner bases (slower but more complete)**
  - Real geometry (very slow but complete)
    - cylindrical algebraic decomposition (Mathematica, Tarski)
    - regular chains (Maple)
- Synthetic methods via axioms (very slow but complete)

# Automated Reasoning Toolset (GeoGebra Discovery)

See also *ATCM 2020*, joint work with Recio



# Many years of joint work

With Botana, Montes, Paez, Vélez, Ladra, Pech and Recio in Santiago, November 2019















# Relation tool and command, 2014

Obtaining the midline theorem

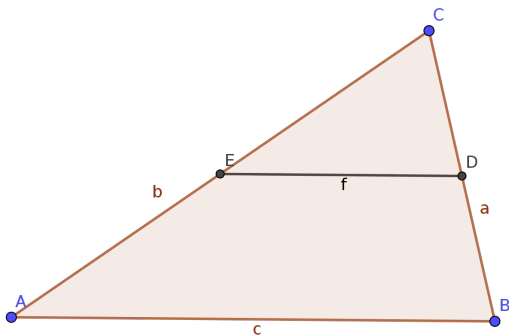

$$a = b$$

**Relation**



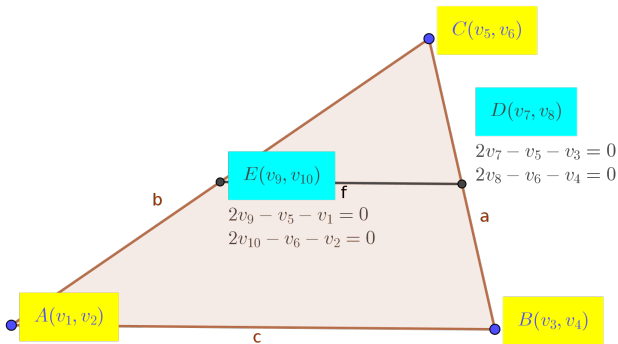
# Relation tool and command, 2014

## Obtaining the midline theorem



# Relation tool and command, 2014

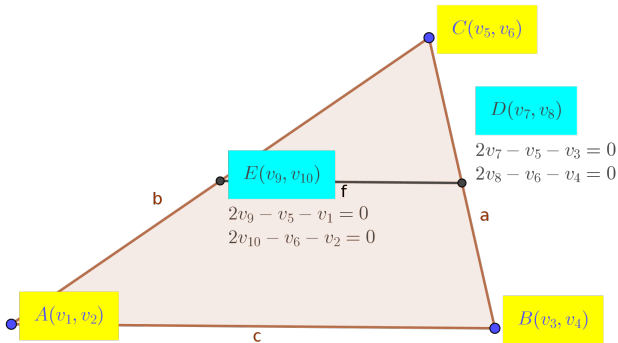
## Obtaining the midline theorem





# Relation tool and command, 2014

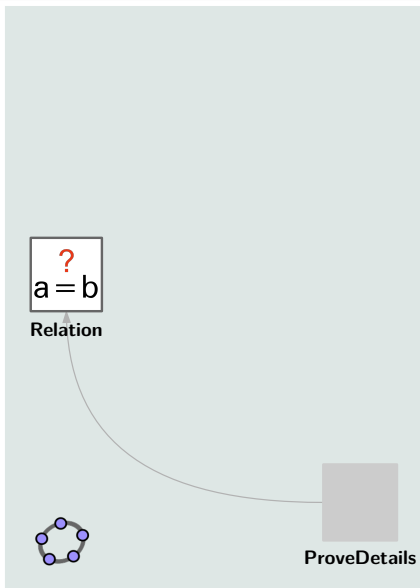
## Obtaining the midline theorem



```
0>> eliminate([-1+2*v8-v6,2*v7-v5,2*v10-v6,2*v9-v5,-1-v11*v9+v11*v7],revlist([v7,v8,v9,v10,v11]))
[1]
// Time 0.01
```

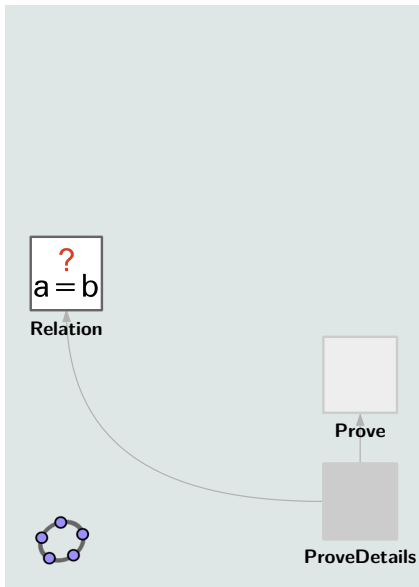
# ProveDetails command, 2012

The diagonals of a square are equally long



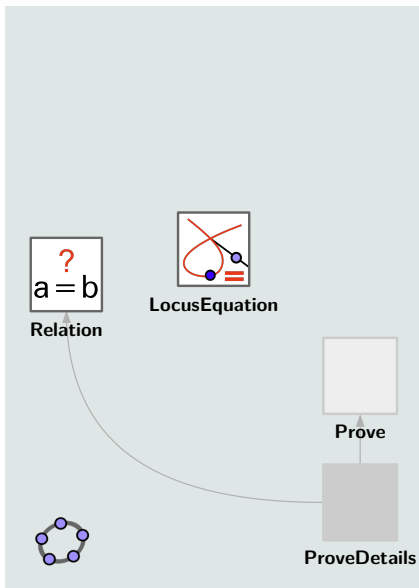
# Prove command

The diagonals of a square are equally long, 2012



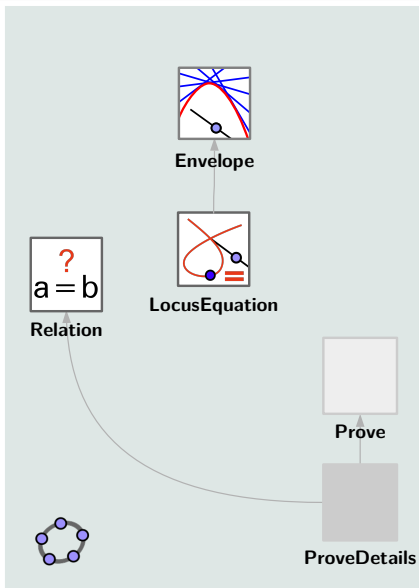
# LocusEquation tool and command, 2010

The moving orthocenter of a set of triangles



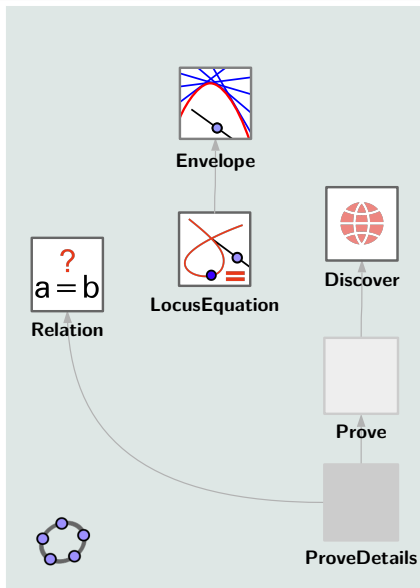
# LocusEquation and Envelope tools and commands, '14

The sliding ladder with a cat in the middle



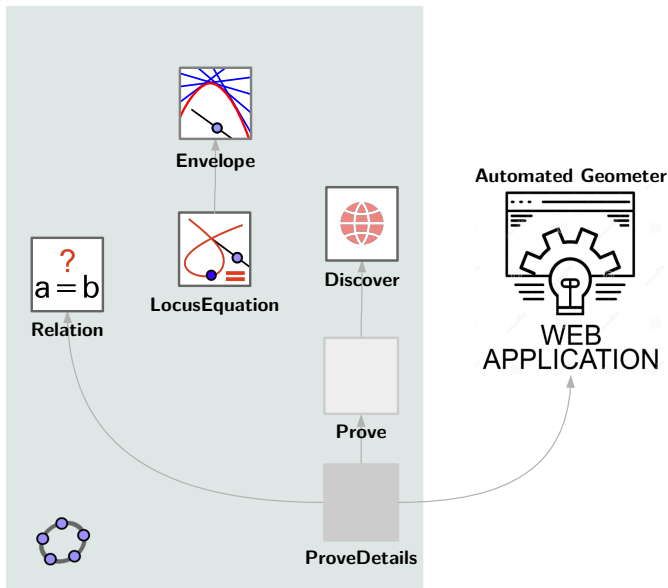
# Discover tool and command, 2020

Remarkable properties of a vertex in a regular pentagon



# Automated Geometer, 2018

Remarkable properties in a regular hexagon; Nine-point circle

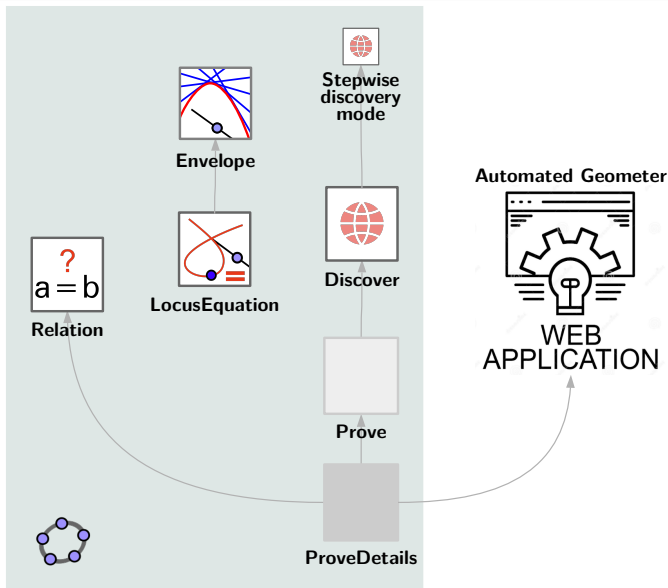


- **OK Geometry** 19.1.1 (Magajna, 2011–, English/Slovenian/Czech): *numerical*
- **Java Geometry Expert** 0.80 (Chou, Gao, Ye, 2008–2009; English/Chinese/German/Italian/Persian/Portuguese translations are available): *symbolic*
- **Géométrie** IV (Gressier, 2014–: French/Spanish/Italian): *numerical*
- **Cabri-géomètre** II (Laborde, 2002–: Chinese/Czech/Danish/Dutch/English/French/German/Hungarian/Italian/Japanese/Korean/Norwegian/Polish/Portuguese/Slovak/Spanish/Swedish/Vietnamese): *numerical*
- **Cinderella** 3.0 (Kortenkamp, Richter-Gebert, 1998–): hidden, *numerical*



# Stepwise discovery mode, 2022, joint work with J.H. Yu

Nine-point circle; Pappus's theorem; Brahmagupta's theorem; ...



# Future work (after some bugfixing)

See *ThEdu* 2021

- Skip trivial properties!

# Future work (after some bugfixing)

See *ThEdu* 2021

- Skip trivial properties! (What is trivial?)

# Future work (after some bugfixing)

See *ThEdu* 2021

- Skip trivial properties! (What is trivial? Whom?)

# Future work (after some bugfixing)

See *ThEdu* 2021

- Skip trivial properties! (What is trivial? Whom?)
- Allow investigating new segments, lines, circles...

# Future work (after some bugfixing)

See *ThEdu* 2021

- Skip trivial properties! (What is trivial? Whom?)
- Allow investigating new segments, lines, circles...
- More control over slow computations (abort, stop/resume)!

# Future work (after some bugfixing)

See *ThEdu* 2021

- Skip trivial properties! (What is trivial? Whom?)
- Allow investigating new segments, lines, circles...
- More control over slow computations (abort, stop/resume)!
- Support angles!

Thank you!

